In[60]:=

```
(*N12*)
(*I can reduce the number of constraints drastically and
 still receive the same output. It seems that certain constraints
 are not as binding as others. The below shall illustrate this.*)

(*N11.1*)
FindInstance[¬ (0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C ≥ 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
0.5 > Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) ≥ 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) ≥ 0.5 && 2 C - B ≤ 1)) && (0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C ≥ 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
0.5 > Max[(2 - C - A) / (2 (2 - 5 A + B)), (C + A) / (2 (5 C - 2 - B))]), {A, B, C}, Reals]
FindInstance[(0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
```

```
A - 4 + 3 B + 4 C ≥ 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
0.5 > Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) ≥ 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) ≥ 0.5 && 2 C - B ≤ 1)) && (0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C ≥ 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
0.5 > Max[(2 - C - A) / (2 (2 - 5 A + B)), (C + A) / (2 (5 C - 2 - B))]), {A, B, C}, Reals]
```

In[62]:=

```
(*N11.2*)
FindInstance[¬ (0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] > Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(C + A) / (2 (5 C - 2 - B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] &&
```

```
¬ ((C - B) / (2 A + 4 C - 6 B) ≥ 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) ≥ 0.5 && 2 C - B ≤ 1)) &&
(0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] > Max[(C + B - 1) / (-4 + 5 C + 3 B), (C + A) / (2 (5 C - 2 - B))]), {A, B, C}, Reals]
FindInstance[(0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] > Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(C + A) / (2 (5 C - 2 - B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) ≥ 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) ≥ 0.5 && 2 C - B ≤ 1)) &&
(0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C ≥ 0 &&
```

```
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] > Max[(C + B - 1) / (-4 + 5 C + 3 B), (C + A) / (2 (5 C - 2 - B))]), {A, B, C}, Reals]
```

In[64]:=

```
(*N11.3*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C ≥ 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A == 0 &&
-4 + 5 C + 3 B > 0 &&
(2 - A - C - 2 B) ≤ 0 &&
Min[0.5] > Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B)),
(C + B - 1) / (-4 + 5 C + 3 B)] &&
¬ (¬ ((2 C - 1) / (9 C - B - 4) ≥ 0.5 && 2 C - B ≤ 1) &&
¬ ((C - B) / (2 A + 4 C - 6 B) ≥ 0.5 && 2 B - C ≥ 0)), {A, B, C}, Reals]
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C ≥ 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A == 0 &&
-4 + 5 C + 3 B > 0 &&
(2 - A - C - 2 B) ≤ 0 &&
Min[0.5] > Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B)),
(C + B - 1) / (-4 + 5 C + 3 B)] &&
¬ ((2 C - 1) / (9 C - B - 4) ≥ 0.5 && 2 C - B ≤ 1) &&
¬ ((C - B) / (2 A + 4 C - 6 B) ≥ 0.5 && 2 B - C ≥ 0), {A, B, C}, Reals]
```

```
(*N11.4*)

(*(C+B-1)/(-4+5C+3B) and (2-A-C-2B)/(2(4-C-3B-4A))
   do not have to be considered separately. (2-C-A)/(2(2-5A+B))<
  0.5=>Max[(C+B-1)/(-4+5C+3B),(2-A-C-2B)/(2(4-C-3B-4A))]<0.5*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
(2 - C - A) / (2 (2 - 5 A + B)) ≤ 0.5 &&
((C + B - 1) / (-4 + 5 C + 3 B) ≥ 0.5 ||
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A)) ≥ 0.5), {A, B, C}, Reals]
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
(2 - C - A) / (2 (2 - 5 A + B)) ≤ 0.5 &&
Max[(C + B - 1) / (-4 + 5 C + 3 B),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] < 0.5, {A, B, C}, Reals]
```

```
(*¬((2C-1)/(9C-B-4)≤Max[(2-C-A)/(2(2-5A+B)),(C+A)/(2(5C-2-B))]&&2C-B≤1)=>0.5>(C+A)/(2(5C-2-B))*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
¬ ((2 C - 1) / (9 C - B - 4) ≤ Max[(2 - C - A) / (2 (2 - 5 A + B)), (C + A) / (2 (5 C - 2 - B))] && 2 C - B ≤ 1) &&
¬ (0.5 > Max[(C + A) / (2 (5 C - 2 - B))]), {A, B, C}, Reals]
```

In[83]:=

```
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
¬ ((2 C - 1) / (9 C - B - 4) ≤ Max[(2 - C - A) / (2 (2 - 5 A + B)), (C + A) / (2 (5 C - 2 - B))] && 2 C - B ≤ 1) &&
0.5 > Max[(C + A) / (2 (5 C - 2 - B))], {A, B, C}, Reals]
```

In[70]:=

```
(*¬((2C-1)/(9C-B-4)≤Max[(2-C-A)/(2(2-5A+B)),(C+A)/(2(5C-2-B))]&&2C-B≤1)=>0.5>(2-C-A)/(2(2-5A+B))*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
¬ ((2 C - 1) / (9 C - B - 4) ≤ Max[(2 - C - A) / (2 (2 - 5 A + B))] && 2 C - B ≤ 1) &&
0.5 ≤ (2 - C - A) / (2 (2 - 5 A + B)), {A, B, C}, Reals]
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A ≥ 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
¬ ((2 C - 1) / (9 C - B - 4) ≤ Max[(2 - C - A) / (2 (2 - 5 A + B))] && 2 C - B ≤ 1) &&
0.5 > (2 - C - A) / (2 (2 - 5 A + B)), {A, B, C}, Reals]
```

```
(*N11.5*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
0.5 > Max[(2 (C + B - 1) (1 - 2 A) 2 - (2 - C - A) (A - 4 + 3 B + 4 C)) /
      (2 ((C - A) (1 - 2 A) 2 - (B - A) (A - 4 + 3 B + 4 C)))(*,
(2(C+B-1)(2C-1)2-(C+A)(A-4+3B+4C))/(2((C-A)(2C-1)2-(C-B)(A-4+3B+4C)))*)] &&
¬ (¬ ((2 (C + B - 1) (A - 4 + 7 C) - (4 C - 2) (A - 4 + 3 B + 4 C)) /
            (2 ((C - A) (A - 4 + 7 C) - (2 C - B - A) (A - 4 + 3 B + 4 C))) ≥
          0.5 && 2 C - B ≤ 1)), {A, B, C}, Reals]

(*(B-A)/(2(4-2C-3A-3B))<
  Max[(C+B-1)/(-4+5C+3B),(2(C+B-1)(1-2A)2-(2-C-A)(A-4+3B+4C))/(2((C-A)(1-2A)2-(B-A)(A-4+3B+4C))),
   (2(C+B-1)(2C-1)2-(C+A)(A-4+3B+4C))/(2((C-A)(2C-1)2-(C-B)(A-4+3B+4C)))] *)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(B - A) / (2 (4 - 2 C - 3 A - 3 B)) > Max[(C + B - 1) / (-4 + 5 C + 3 B),
(2 (C + B - 1) (1 - 2 A) 2 - (2 - C - A) (A - 4 + 3 B + 4 C)) /
      (2 ((C - A) (1 - 2 A) 2 - (B - A) (A - 4 + 3 B + 4 C))),
(2 (C + B - 1) (2 C - 1) 2 - (C + A) (A - 4 + 3 B + 4 C)) / (2 ((C - A) (2 C - 1) 2 - (C - B) (A - 4 + 3 B + 4 C)))],
 {A, B, C}, Reals]
```

In[84]:=

```
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(B - A) / (2 (4 - 2 C - 3 A - 3 B)) ≤ Max[(C + B - 1) / (-4 + 5 C + 3 B),
(2 (C + B - 1) (1 - 2 A) 2 - (2 - C - A) (A - 4 + 3 B + 4 C)) /
      (2 ((C - A) (1 - 2 A) 2 - (B - A) (A - 4 + 3 B + 4 C))),
(2 (C + B - 1) (2 C - 1) 2 - (C + A) (A - 4 + 3 B + 4 C)) / (2 ((C - A) (2 C - 1) 2 - (C - B) (A - 4 + 3 B + 4 C)))],
 {A, B, C}, Reals]
```

In[77]:=

```
(*(2(C+B-1)(4-2C-3B-3A)-(2-A-C-3B)(A-4+3B+4C))/(2((C-A)(4-2C-3B-3A)-(C-A)(A-4+3B+4C)))<0.5*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(2 (C + B - 1) (4 - 2 C - 3 B - 3 A) - (2 - A - C - 3 B) (A - 4 + 3 B + 4 C)) /
     (2 ((C - A) (4 - 2 C - 3 B - 3 A) - (C - A) (A - 4 + 3 B + 4 C))) ≥ 0.5, {A, B, C}, Reals]
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(2 (C + B - 1) (4 - 2 C - 3 B - 3 A) - (2 - A - C - 3 B) (A - 4 + 3 B + 4 C)) /
     (2 ((C - A) (4 - 2 C - 3 B - 3 A) - (C - A) (A - 4 + 3 B + 4 C))) < 0.5, {A, B, C}, Reals]
```

```
(*(2(C+B-1)(A+3C-4B)-(2C-2B)(A-4+3B+4C))/(2((C-A)(A+3C-4B)-(A-2B+C)(A-4+3B+4C)))<0.5&&
 0.5>Max[(2(C+B-1)(2C-1)2-(C+A)(A-4+3B+4C))/(2((C-A)(2C-1)2-(C-B)(A-4+3B+4C)))]
   if (C towards 0) is relevant. *)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(2 (C + B - 1) (A + 3 C - 4 B) - (2 C - 2 B) (A - 4 + 3 B + 4 C)) /
     (2 ((C - A) (A + 3 C - 4 B) - (A - 2 B + C) (A - 4 + 3 B + 4 C))) ≥
   0.5 > Max[(2 (C + B - 1) (2 C - 1) 2 - (C + A) (A - 4 + 3 B + 4 C)) /
      (2 ((C - A) (2 C - 1) 2 - (C - B) (A - 4 + 3 B + 4 C)))] && 2 B - C ≥ 0, {A, B, C}, Reals]
```

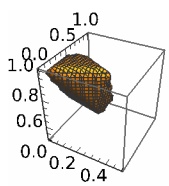
In[85]:=

```
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(2 (C + B - 1) (A + 3 C - 4 B) - (2 C - 2 B) (A - 4 + 3 B + 4 C)) /
     (2 ((C - A) (A + 3 C - 4 B) - (A - 2 B + C) (A - 4 + 3 B + 4 C))) ≤ 0.5 &&
  0.5 > Max[(2 (C + B - 1) (2 C - 1) 2 - (C + A) (A - 4 + 3 B + 4 C)) /
      (2 ((C - A) (2 C - 1) 2 - (C - B) (A - 4 + 3 B + 4 C)))] && 2 B - C ≥ 0, {A, B, C}, Reals]
```

In[81]:=

```
(*(2(C+B-1)(A-4+7C)-(4C-2)(A-4+3B+4C))/(2((C-A)(A-4+7C)-(2C-B-A)(A-4+3B+4C)))<
 0.5 if (A towards 1) is relevant.*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(2 (C + B - 1) (A - 4 + 7 C) - (4 C - 2) (A - 4 + 3 B + 4 C)) /
    (2 ((C - A) (A - 4 + 7 C) - (2 C - B - A) (A - 4 + 3 B + 4 C))) > 0.5 && 2 C - B ≤ 1, {A, B, C}, Reals]
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
(2 (C + B - 1) (A - 4 + 7 C) - (4 C - 2) (A - 4 + 3 B + 4 C)) /
    (2 ((C - A) (A - 4 + 7 C) - (2 C - B - A) (A - 4 + 3 B + 4 C))) ≤ 0.5 && 2 C - B ≤ 1, {A, B, C}, Reals]
```

Null



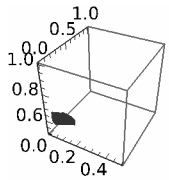

```
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A>0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B>0

not possible*)
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
```

```
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
Min[(C + B - 1) / (-4 + 5 C + 3 B), 0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(C+A)/(2(5C-2-B)),
(2-A-C-2B)/(2(4-C-3B-4A))]
&&¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*)]
$Aborted
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A>0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B<0

This constellation is one of the few possible ones. The calculation of
     "Reduce[]" is too work intensive for my version of mathematica. The
     "RegionPlot3D" shows that the constellation is possible:*)
(*N11.2*)
r2 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(C + A) / (2 (5 C - 2 - B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
```

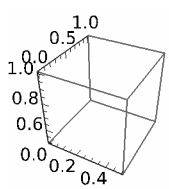
```
¬ ((2 C - 1) / (9 C - B - 4) > 0.5 && 2 C - B ≤ 1), {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]
```




```
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A>0&&
A-4+7C<0&&
4-C-3B-4A>0&&
-4+5C+3B<0

This constellation is one of the few possible ones. The
     calculation of "Reduce[]" is too work intensive for my version
     of mathematica. The "RegionPlot3D" shows that the constellation
     is possible but at least one of the parameters has to equal some
     exact value (the plot does not show any mass). "FindInstance[]"
     shows that there is indeed a solution:*)
(*N11.3*)
FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(C + A) / (2 (5 C - 2 - B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) < Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + B - 1) / (-4 + 5 C + 3 B),
```

```
(C + A) / (2 (5 C - 2 - B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] && 2 C - B ≤ 1), {A, B, C}, Reals]
r3 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(C + A) / (2 (5 C - 2 - B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) < Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + B - 1) / (-4 + 5 C + 3 B),
(C + A) / (2 (5 C - 2 - B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] && 2 C - B ≤ 1),
  {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]
{{A → 0.05555555555555557`, B → 0.09126984126984128`, C → 0.5545634920634922`}}
```




```
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B<0

not possible*)
```

```
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5, (2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(C+B-1)/(-4+5C+3B),
(C+A)/(2(5C-2-B))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*)]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
4-C-3B-4A<0&&
-4+5C+3B<0

Not possible. The calculation of "Reduce[]" is too work intensive for my version
     of mathematica. "FindInstance[]" shows that there is indeed no solution:*)

FindInstance[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A < 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A)),
(C + B - 1) / (-4 + 5 C + 3 B),
```

```
(C + A) / (2 (5 C - 2 - B))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) > 0.5 && 2 C - B ≤ 1), {A, B, C}, Reals]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B<0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5(*,(C+B-1)/(-4+5C+3B),(2-A-C-2B)/(2(4-C-3B-4A))*)] ≥
   Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(C+A)/(2(5C-2-B))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*)]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
4-C-3B-4A<0&&
-4+5C+3B<0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
```

```
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A < 0 &&
-4 + 5 C + 3 B < 0 &&
Min[(C + B - 1) / (-4 + 5 C + 3 B), 0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(2-A-C-2B)/(2(4-C-3B-4A)),
(C+A)/(2(5C-2-B))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*)]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C<0&&
4-C-3B-4A>0&&
-4+5C+3B<0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5, (2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(C+B-1)/(-4+5C+3B),
(C+A)/(2(5C-2-B))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)<Max[1/6,(2-C-A)/(2(2-5A+B)),
(C+B-1)/(-4+5C+3B),
```

```
(C+A)/(2(5C-2-B))]&&2C-B≤1)*)]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C<0&&
4-C-3B-4A<0&&
-4+5C+3B<0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A < 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(2-A-C-2B)/(2(4-C-3B-4A)),
(C+B-1)/(-4+5C+3B),
(C+A)/(2(5C-2-B))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)<Max[1/6,(2-C-A)/(2(2-5A+B)),
(2-A-C-2B)/(2(4-C-3B-4A)),
(C+B-1)/(-4+5C+3B),
(C+A)/(2(5C-2-B))]&&2C-B≤1)*)]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C<0&&
4-C-3B-4A>0&&
-4+5C+3B<0

not possible*)
```

```
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5, (C + B - 1) / (-4 + 5 C + 3 B), (2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] ≥ Max[1 / 6]
(*,(2-C-A)/(2(2-5A+B)),
(C+A)/(2(5C-2-B))&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)<Max[1/6,(2-C-A)/(2(2-5A+B)),
(2-A-C-2B)/(2(4-C-3B-4A)),
(C+B-1)/(-4+5C+3B),
(C+A)/(2(5C-2-B))]&&2C-B≤1)*)]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0&&
A-4+7C<0&&
4-C-3B-4A<0&&
-4+5C+3B<0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A < 0 &&
-4 + 5 C + 3 B < 0 &&
Min[(C + B - 1) / (-4 + 5 C + 3 B), 0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
```

```
(2-A-C-2B)/(2(4-C-3B-4A)),
(C+A)/(2(5C-2-B))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)<Max[1/6,(2-C-A)/(2(2-5A+B)),
(2-A-C-2B)/(2(4-C-3B-4A)),
(C+B-1)/(-4+5C+3B),
(C+A)/(2(5C-2-B))]&&2C-B≤1)*)]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B>0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
Min[0.5,
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(C+A)/(2(5C-2-B)),
(C+B-1)/(-4+5C+3B)]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*)]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
```

```
4-C-3B-4A<0&&
-4+5C+3B>0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A < 0 &&
-4 + 5 C + 3 B > 0 &&
Min[0.5] ≥ Max[(2 - C - A) / (2 (2 - 5 A + B)), (2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))]]
(*1/6,(2-C-A)/(2(2-5A+B)),(C+A)/(2(5C-2-B)),(C+B-1)/(-4+5C+3B),
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*)
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A<0&&
A-4+7C<0&&
4-C-3B-4A>0&&
-4+5C+3B>0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
Min[0.5,
```

```
(2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))](*,
(C+A)/(2(5C-2-B)),
(C+B-1)/(-4+5C+3B)]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)<Max[1/6,(2-C-A)/(2(2-5A+B)),
(C+A)/(2(5C-2-B)),
(C+B-1)/(-4+5C+3B),
(2-A-C-2B)/(2(4-C-3B-4A))]&&2C-B≤1)*)]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A<0&&
A-4+7C<0&&
4-C-3B-4A<0&&
-4+5C+3B>0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A < 0 &&
-4 + 5 C + 3 B > 0 &&
Min[0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B))]]
(*,(C+A)/(2(5C-2-B)),
(C+B-1)/(-4+5C+3B),
(2-A-C-2B)/(2(4-C-3B-4A))]]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)<Max[1/6,(2-C-A)/(2(2-5A+B)),
(C+A)/(2(5C-2-B)),
(C+B-1)/(-4+5C+3B),
(2-A-C-2B)/(2(4-C-3B-4A))]&&2C-B≤1)]*)
(*When considering -4+5C+3B==0 and 4-C-3B-4A==0,
I can restrict myself to the below constellations. Any other
  constellation of inequalities regarding A-4+3B+4C, 4-C-3B-4A,
```

```
4-2C-3B-3A and -4+5C+3B makes the constellation non-admissible.*)
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
-4 + 5 C + 3 B == 0]
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 3 B + 4 C > 0 &&
4 - C - 3 B - 4 A == 0 &&
-4 + 5 C + 3 B > 0]
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A>0&&
A-4+7C<0&&
4-C-3B-4A>0&&
-4+5C+3B==0

not possible*)
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B == 0 &&
Min[0.5, (2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
```

```
(C + A) / (2 (5 C - 2 - B))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) < Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B))] && 2 C - B ≤ 1)]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A>0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B==0

not possible*)

Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B == 0 &&
Min[0.5, (2 - A - C - 2 B) / (2 (4 - C - 3 B - 4 A))] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B))] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) > 0.5 && 2 C - B ≤ 1)]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A<0&&
A-4+7C<0&&
4-C-3B-4A==0&&
-4+5C+3B>0

not possible*)
```

```
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A == 0 &&
-4 + 5 C + 3 B > 0 &&
Min[0.5] ≥ Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B)),
(C + B - 1) / (-4 + 5 C + 3 B)] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) < Max[1 / 6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B)),
(C + B - 1) / (-4 + 5 C + 3 B)] && 2 C - B ≤ 1)]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
4-C-3B-4A==0&&
-4+5C+3B>0

possible*)
(*N11.4*)
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A == 0 &&
-4 + 5 C + 3 B > 0 &&
```

```
Min[0.5] ≥ Max[1/6, (2 - C - A)/(2 (2 - 5 A + B)),
(C + A)/(2 (5 C - 2 - B)),
(C + B - 1)/(-4 + 5 C + 3 B)] &&
¬ ((C - B)/(2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1)/(9 C - B - 4) > 0.5 && 2 C - B ≤ 1)]
```

··· **Reduce:**

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

{}

··· **Reduce:**

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

··· **Reduce:**

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

··· **Reduce:**

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

··· **Reduce:**

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

··· **Reduce:**

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

··· **Reduce:**

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

Reduce:

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

Reduce:

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

Reduce:

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

Reduce:

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

$$\left(\left(0 < A \le \frac{3}{19} \;\&\&\; \frac{1}{13}(8-3A) < C < \frac{1}{5}(4-3A)\right) \| \left(\frac{3}{19} < A < \frac{3}{11} \;\&\&\; \frac{1+A}{2} < C < \frac{1}{5}(4-3A)\right)\right) \;\&\&$$
$$B == \frac{1}{3}(4-5C)$$

$$\left(\left(\frac{3}{7} < B \le \frac{4}{9} \;\&\&\; 4-7B < C < 1\right) \| \left(\frac{4}{9} < B \le \frac{4}{7} \;\&\&\; \frac{1}{3}(4-3B) < C < 1\right) \| \left(\frac{4}{7} < B < \frac{7}{11} \;\&\&\; \frac{1}{3}(-4+11B) < C < 1\right)\right) \;\&\&\; A == \frac{1}{4}(4-3B-C)$$

Reduce:

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

Reduce:

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

Reduce:

"Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result."

False

```
··· Reduce:
 "Reduce was unable to solve the system with inexact coefficients. The answer was
   obtained by solving a corresponding exact system and numericizing the result."
((0.2727272727272727` < A ≤ 0.35294117647058826` && 0.2` (8.` - 11.` A) < C < 1.`) ||
   (0.35294117647058826` < A < 0.375` && -2.` + 8.` A ≤ C < 1.`)) &&
 B == 0.3333333333333333` (4.` - 4.` A - 1.` C)
r4 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A == 0 &&
-4 + 5 C + 3 B > 0 &&
Min[0.5] ≥ Max[1/6, (2 - C - A) / (2 (2 - 5 A + B)),
(C + A) / (2 (5 C - 2 - B)),
(C + B - 1) / (-4 + 5 C + 3 B)] &&
¬ ((C - B) / (2 A + 4 C - 6 B) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 C - 1) / (9 C - B - 4) > 0.5 && 2 C - B ≤ 1), {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]
(*lowest upper bound on q:  (2(C+B-1-(C-A)rho))/(A-4+3B+4C)
other constraints:
A-4+3B+4C<0&&
-4+5C+3B<0
note: the sign of A-4+7C does not matter,
as the direction of inequalities remains the same in both cases.

This constellation is one of the few possible ones. The calculation
 of "Reduce[]" is too work intensive for my version of mathematica. The
 "RegionPlot3D" shows that the constellation is possible:*)
(*N11.5*)
r5 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
```

```
-4 + 5 C + 3 B < 0 &&
0.5 ≥ Max[(B - A) / (2 (4 - 2 C - 3 A - 3 B)), (C + B - 1) / (-4 + 5 C + 3 B),
(2 (C + B - 1) (1 - 2 A) 2 - (2 - C - A) (A - 4 + 3 B + 4 C)) /
      (2 ((C - A) (1 - 2 A) 2 - (B - A) (A - 4 + 3 B + 4 C))),
(2 (C + B - 1) (2 C - 1) 2 - (C + A) (A - 4 + 3 B + 4 C)) / (2 ((C - A) (2 C - 1) 2 - (C - B) (A - 4 + 3 B + 4 C))),
(2 (C + B - 1) (4 - 2 C - 3 B - 3 A) - (2 - A - C - 3 B) (A - 4 + 3 B + 4 C)) /
       (2 ((C - A) (4 - 2 C - 3 B - 3 A) - (C - A) (A - 4 + 3 B + 4 C))) &&
¬ ((2 (C + B - 1) (A + 3 C - 4 B) - (2 C - 2 B) (A - 4 + 3 B + 4 C)) /
            (2 ((C - A) (A + 3 C - 4 B) - (A - 2 B + C) (A - 4 + 3 B + 4 C))) > 0.5 && 2 B - C ≥ 0) &&
¬ ((2 (C + B - 1) (A - 4 + 7 C) - (4 C - 2) (A - 4 + 3 B + 4 C)) /
            (2 ((C - A) (A - 4 + 7 C) - (2 C - B - A) (A - 4 + 3 B + 4 C))) > 0.5 &&
         2 C - B ≤ 1)], {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]

(*lowest upper bound on q:  (2(C+B-1-(C-A)rho))/(A-4+3B+4C)
other constraints:
A-4+3B+4C<0&&
-4+5C+3B>0
note: the sign of A-4+7C does not matter,
as the direction of inequalities remains the same in both cases;
as (C+B-1)/(-4+5C+3B)<0.5 in any case in this parameter constellation,
I can remove the "Min".

not possible*)
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B > 0 &&
(C + B - 1) / (-4 + 5 C + 3 B) > 0.5]
Reduce[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
```

```
-4 + 5 C + 3 B > 0 &&
(C + B - 1) / (-4 + 5 C + 3 B) ≥ (B - A) / (2 (4 - 2 C - 3 A - 3 B))]
(*&&
(C+B-1)/(-4+5C+3B)≥(2(C+B-1)(1-2A)2-(2-C-A)(A-4+3B+4C))/(2((C-A)(1-2A)2-(B-A)(A-4+3B+4C)))&&
(C+B-1)/(-4+5C+3B)≥(2(C+B-1)(2C-1)2-(C+A)(A-4+3B+4C))/(2((C-A)(2C-1)2-(C-B)(A-4+3B+4C)))&&
(C+B-1)/(-4+5C+3B)≥
   (2(C+B-1)(4-2C-3B-3A)-(2-A-C-3B)(A-4+3B+4C))/(2((C-A)(4-2C-3B-3A)-(C-A)(A-4+3B+4C)))&&
¬((2(C+B-1)(A+3C-4B)-(2C-2B)(A-4+3B+4C))/(2((C-A)(A+3C-4B)-(A-2B+C)(A-4+3B+4C)))>
      Min[(C+B-1)/(-4+5C+3B),0.5]&&2B-C≥0)&&
¬((2(C+B-1)(A-4+7C)-(4C-2)(A-4+3B+4C))/(2((C-A)(A-4+7C)-(2C-B-A)(A-4+3B+4C)))>
      Min[(C+B-1)/(-4+5C+3B),0.5]&&2C-B≤1)*)
(*lowest upper bound on q:  (2(C+B-1-(C-A)rho))/(A-4+3B+4C)
other constraints:
A-4+3B+4C<0&&
-4+5C+3B=0

not possible*)

Reduce[(2 (C + B - 1 - (C - A) rho)) / (A - 4 + 3 B + 4 C) < 2 rho &&
-4 + 5 C + 3 B == 0 &&
0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
0 < rho < 0.5]
(*lowest upper bound on q:  (2(C+B-1-(C-A)rho))/(A-4+3B+4C)
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A<0

not possible*)

Reduce[(2 (C + B - 1 - (C - A) rho)) / (A - 4 + 3 B + 4 C) < 2 rho &&
0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
A - 4 + 3 B + 4 C < 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
```

```
4 - 2 C - 3 B - 3 A < 0 &&
0 < rho < 0.5]
(*The final plot is the following:*)
Show[{r1, r2, r3, r4, r5}, AxesLabel → {α, β, γ}]
(*N12*)
(*I can reduce the number of constraints drastically and still
 receive the same output. It seems that certain constraints are not
 as binding as others. The only important ones seem to be (A to γ),
(B to α) and (B to γ). Note that the below is no proof of this statement.*)
(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A>0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B>0

This constellation is one of the few possible ones. The calculation of
     "Reduce[]" is too work intensive for my version of mathematica. The
     "RegionPlot3D" shows that the constellation is possible:*)
R1 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B > 0 &&
0.5 ≥ Max[(2 - C - A) / (2 (2 - 5 A + B))](*,1/6,
(C+A)/(2(5C-2-B)),
(C+B-1)/(-4+5C+3B),
(2-A-C-2B)/(2(4-C-3B-4A))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*), {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]

(*lowest upper bound on q: 2rho
other constraints:
```

```
A-4+3B+4C<0&&
4-2C-3B-3A>0&&
A-4+7C>0&&
4-C-3B-4A>0&&
-4+5C+3B<0

This constellation is one of the few possible ones. The calculation of
     "Reduce[]" is too work intensive for my version of mathematica. The
     "RegionPlot3D" shows that the constellation is possible:*)
R2 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
Min[0.5] ≥ Max[(C + B - 1) / (-4 + 5 C + 3 B), (C + A) / (2 (5 C - 2 - B))](*,
(2-C-A)/(2(2-5A+B)),1/6,
(2-A-C-2B)/(2(4-C-3B-4A))]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*), {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C<0&&
4-2C-3B-3A>0&&
A-4+7C<0&&
4-C-3B-4A>0&&
-4+5C+3B<0

This constellation is one of the few possible ones. The
     calculation of "Reduce[]" is too work intensive for my version
     of mathematica. The "RegionPlot3D" shows that the constellation
     is possible but at least one of the parameters has to equal some
     exact value (the plot does not show any mass). "FindInstance[]"
     shows that there is indeed a solution:*)
```

```
R3 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
4 - 2 C - 3 B - 3 A > 0 &&
A - 4 + 7 C < 0 &&
4 - C - 3 B - 4 A > 0 &&
-4 + 5 C + 3 B < 0 &&
(*Min[0.5]≥Max[(2-C-A)/(2(2-5A+B)),
(C+B-1)/(-4+5C+3B),1/6,
(C+A)/(2(5C-2-B)),
(2-A-C-2B)/(2(4-C-3B-4A))]&&*)
(*¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&*)
¬ ((2 C - 1) / (9 C - B - 4) < Max[(2 - C - A) / (2 (2 - 5 A + B))](*,
1/6,(C+B-1)/(-4+5C+3B),
(C+A)/(2(5C-2-B)),
(2-A-C-2B)/(2(4-C-3B-4A))]*) && 2 C - B ≤ 1), {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]

(*lowest upper bound on q: 2rho
other constraints:
A-4+3B+4C>0&&
4-2C-3B-3A<0&&
A-4+7C>0&&
4-C-3B-4A==0&&
-4+5C+3B>0

possible*)

R4 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C > 0 &&
4 - 2 C - 3 B - 3 A < 0 &&
A - 4 + 7 C > 0 &&
4 - C - 3 B - 4 A == 0 &&
-4 + 5 C + 3 B > 0 &&
```

```
Min[0.5] ≥ Max[(2 - C - A) / (2 (2 - 5 A + B))](*,(C+A)/(2(5C-2-B)),
(C+B-1)/(-4+5C+3B),
1/6]&&
¬((C-B)/(2A+4C-6B)>0.5&&2B-C≥0)&&
¬((2C-1)/(9C-B-4)>0.5&&2C-B≤1)*), {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]

(*lowest upper bound on q:  (2(C+B-1-(C-A)rho))/(A-4+3B+4C)
other constraints:
A-4+3B+4C<0&&
-4+5C+3B<0
note: the sign of A-4+7C does not matter,
as the direction of inequalities remains the same in both cases.

This constellation is one of the few possible ones. The calculation
 of "Reduce[]" is too work intensive for my version of mathematica. The
 "RegionPlot3D" shows that the constellation is possible:*)

R5 = RegionPlot3D[0 < A < B < C < 1 && B - A < C - B &&
2 C - 1 > 0 &&
1 - 2 A > 0 &&
2 A - C < 0 &&
2 C - A > 1 &&
A - 4 + 3 B + 4 C < 0 &&
-4 + 5 C + 3 B < 0 &&
0.5 ≥ Max[(2 (C + B - 1) (1 - 2 A) 2 - (2 - C - A) (A - 4 + 3 B + 4 C)) /
        (2 ((C - A) (1 - 2 A) 2 - (B - A) (A - 4 + 3 B + 4 C))), (C + B - 1) / (-4 + 5 C + 3 B),
(2 (C + B - 1) (2 C - 1) 2 - (C + A) (A - 4 + 3 B + 4 C)) / (2 ((C - A) (2 C - 1) 2 - (C - B) (A - 4 + 3 B + 4 C)))]
       (*(B-A)/(2(4-2C-3A-3B)),
(2(C+B-1)(4-2C-3B-3A)-(2-A-C-3B)(A-4+3B+4C))/(2((C-A)(4-2C-3B-3A)-(C-A)(A-4+3B+4C)))&&
¬((2(C+B-1)(A+3C-4B)-(2C-2B)(A-4+3B+4C))/(2((C-A)(A+3C-4B)-(A-2B+C)(A-4+3B+4C)))>0.5&&2B-C≥0)&&
¬((2(C+B-1)(A-4+7C)-(4C-2)(A-4+3B+4C))/(2((C-A)(A-4+7C)-(2C-B-A)(A-4+3B+4C)))>0.5&&2C-B≤1)]*),
   {A, 0, 0.5}, {B, 0, 1}, {C, 0.5, 1}]
```